Measuring structural efficiency in bridges' sketches

or

Lo que cunde un año en CIMNE Barcelona

Mariano Vázquez Espí

Barcelona, Jan 18, 2012[1]

Spanish Abstract

¿Puede predecirse el rendimiento estructural que tendrá un puente con sólo analizar los primeros bocetos de su diseñador? La respuesta es afirmativa... con algunas condiciones: a) suscribir el axioma fundamental sobre contabilidad de la termodinámica clásica, b) limitarse a aquellos costes que sean funciones lineales de los volúmenes de tensión en tracción y en compresión de la estructura, y c) distinguir exquisitamente entre rigor y precisión. Veremos que las dos primeras condiciones no son difíciles de cumplir mediante algunos ejemplos: desde el bien conocido arco parabólico hasta un puente de dos kilómetros, el *Akashi Kaikyo Bridge* en Japón. El recorrido por los puentes nos llevará también de excursión por parcelas termodinámicas (con Clausius y Georgescu-Roegen como artistas invitados) y de filosofía de la ciencia (con los editores de los Discorsi de Galileo Galilei y Feyerabend), parcelas que, como se dice, son de "rabiosa actualidad" para entender algo de la actual crisis financiera (sí, lector o lectora, has leído bien). Aunque este *Coffee* es continuación de otro el pasado 25 de mayo, no es necesario que el público tenga nociones previas, salvo las fundamentales de análisis estructural. La traca final constará del enunciado de algunos *open problems* y una única petición final del autor.

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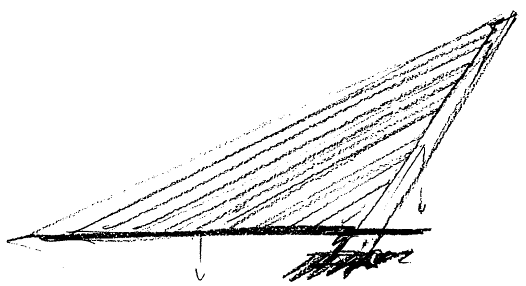
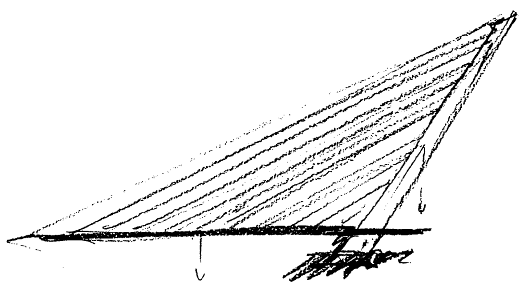


Today, the most popular architects are Mediterranean designers who are unusually young. A case in point is the Valencian Santiago Calatrava, who commutes between Zurich and Paris, and works between engineering and sculpture, possibly in the tradition of Felix Candela and definitely in that of Gaudi.

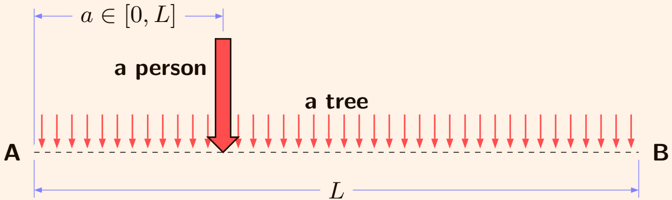
Fernández-Galiano [1992][2]

Can the structural efficiency be predicted from a bridge sketch at the very beginning of design process?



More precisely: can we compare the structural efficiency of two different sketches for the same bridge, only with information about their shapes and without any information about materials and their physical properties?

The design problem of a bridge

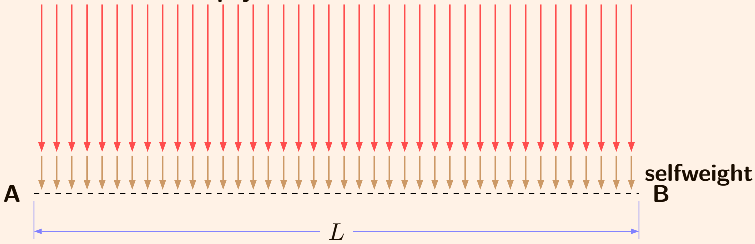


A person, a tree

> [...] un hombre lanza un árbol entre dos piedras para pasar un río. Hasta el siglo XVI [*sic*] no se planteó la representación de la fuerza con un vector y el primer antifunicular de cargas es de mediado el siglo XVIII [*sic*]
>
> MANTEROLA (1997:viii)

The design problem of a bridge

An useful uniform load, a constant cross-section beam

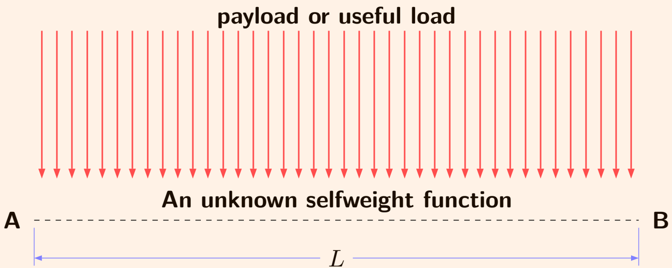


(Isomorphism case: useful load and selfweight have the same shape)

> [...] un hombre lanza un árbol entre dos piedras para pasar un río. Hasta el siglo XVI [*sic*] no se planteó la representación de la fuerza con un vector y el primer antifunicular de cargas es de mediado el siglo XVIII [*sic*]
>
> MANTEROLA (1997:viii)

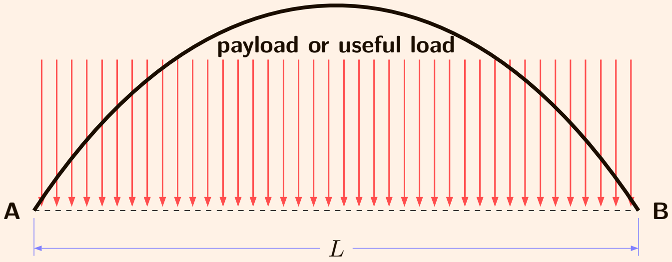
The design problem of a bridge

An useful uniform load, a bridge of unknown shape



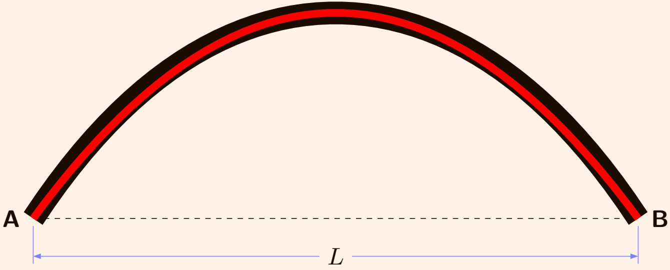
> [. . .] un hombre lanza un árbol entre dos piedras para pasar un río. Hasta el siglo XVI [*sic*] no se planteó la representación de la fuerza con un vector y el primer antifunicular de cargas es de mediado el siglo XVIII [*sic*]
>
> Manterola (1997:viii)

Parabola and Catenary Arches



Only useful, uniform load will be considered in the sequel for the shake of simplicity. But of course several others load hypothesis have to be considered, perhaps most important... The selfweight will be considered very small, or estimated by Galileo's rule (see below)

Parabola and Catenary Arches

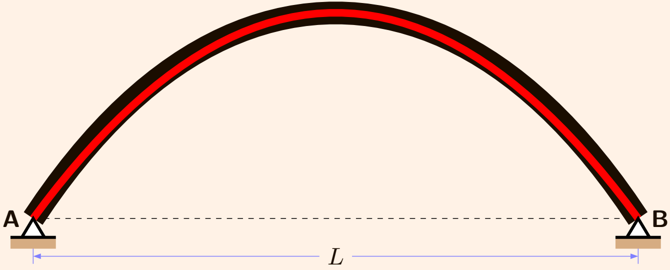


The parabola-like arches are well known to be used since ca. 1250BC (*Ramesseum* stores, West Bank, Luxor).

HOOKE (1671) announced that catenary is the optimal solution for an arch.

GALILEO (1638) noted that catenary and parabola are very similar when curvature is not very great. In 1691 LEIBNIZ, HUYGENS, and Johann BERNOULLI derived simultaneously the catenary equation in response to a challenge by Jakob BERNOULLI. The difference between the two equations is surprisingly great in spite of their graphs closely approximate to each other as the curvature gets smaller and is almost exact when the elevation is less than 45^o, as it was noted by GALILEO.

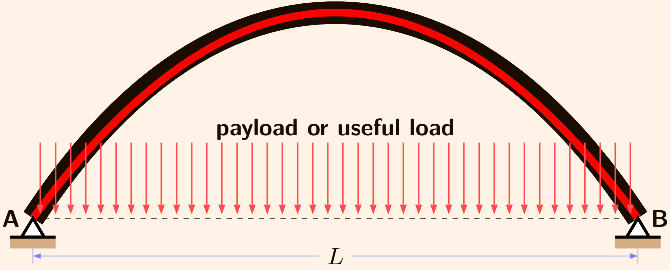
Parabola and Catenary Arches



Nowadays, it is well known that a parabola is the funicular curve for a weight function that is constant along horizontal line, whilst a catenary does for constant function along the curve itself. Hence, we could hope that a mixed curve will be the funicular for a mixed load.

Note that we need two fixed supports for equilibrium condition.

Parabola and Catenary Arches

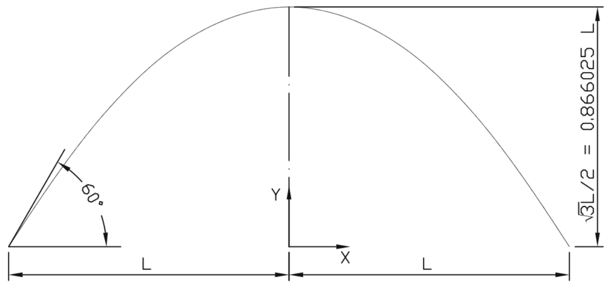


There is another problem with the useful load: we must ‘transmit’ it to the arch…

…and there are very imaginative, fancy solutions for this problem…

Parab

Prager structures: the loads height is free of cost...



Optimal parabolic arch, (after Rozvany and Prager 1979)

... or "optimized" (After Darwich, Gilbert & Tyas (2010)):

	$\sigma^-/\sigma^+ = 3$	$\sigma^-/\sigma^+ = 2$	$\sigma^-/\sigma^+ = 1$	$\sigma^-/\sigma^+ = 2/3$	$\sigma^-/\sigma^+ = 1/2$
Layout					
Volume	0.577684	0.577656	0.575672	0.572634	0.569728
%diff	0.0578%	0.0530%	-0.2907%	-0.8169%	-1.320%
Approx. θ_l	30°	35°	45°	50°	55°
Analytic. θ_l	30°	35.26°	45°	50.77°	54.74°

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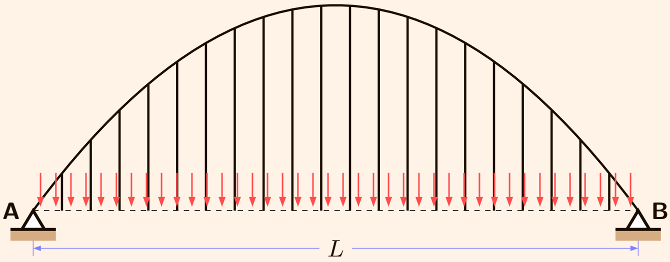
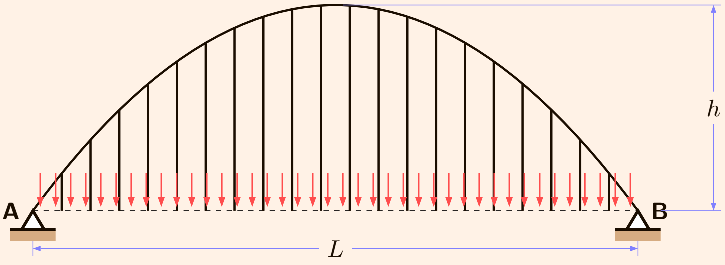
Parabola and Catenary Arches



...but the traditional solution for transmitting the load to the arch has been to use vertical hangers. **We wish to cross the river not to go up a mountain!**

Vertical hangers or another artifact must be required to take the load from **AB** line in a rational formulation of the problem.

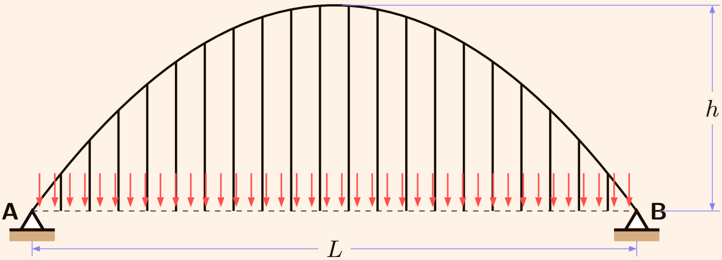
Parabola and Catenary Arches

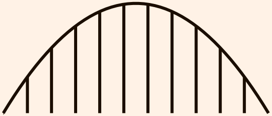


This completes the definition of a structural scheme that only depends on height h.

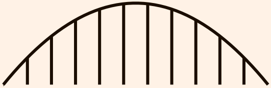
We can proceed with the problem “as is” just at this moment. Simply we just have to solve $\min_h \mathcal{C}$ for the interesting cost $\mathcal{C}$, e.g., material volume V...

Parabola and Catenary Arches

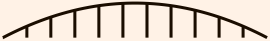




$V_{\mathrm{opt}} \propto 6.06$ if $\mathbf{f}^{+}/\mathbf{f}^{-} = 10$



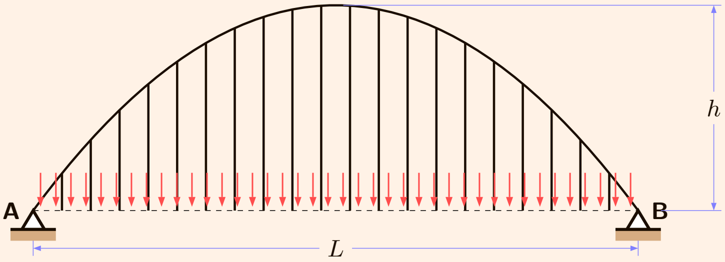
$V_{\mathrm{opt}} \propto 0.816$ if $\mathbf{f}^{+}/\mathbf{f}^{-} = 1$

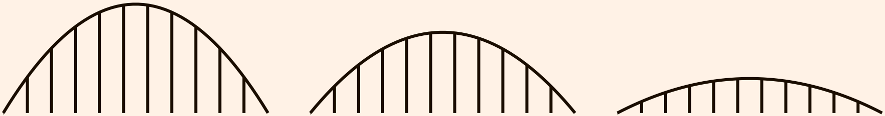


$V_{\mathrm{opt}} \propto 0.191$ if $\mathbf{f}^{+}/\mathbf{f}^{-} = 0.1$

The conclusion with this approach is that the answer to the main question must be: **“No, because the structural efficiency of a given shape depends strongly on material properties”**.

Parabola and Catenary Arches



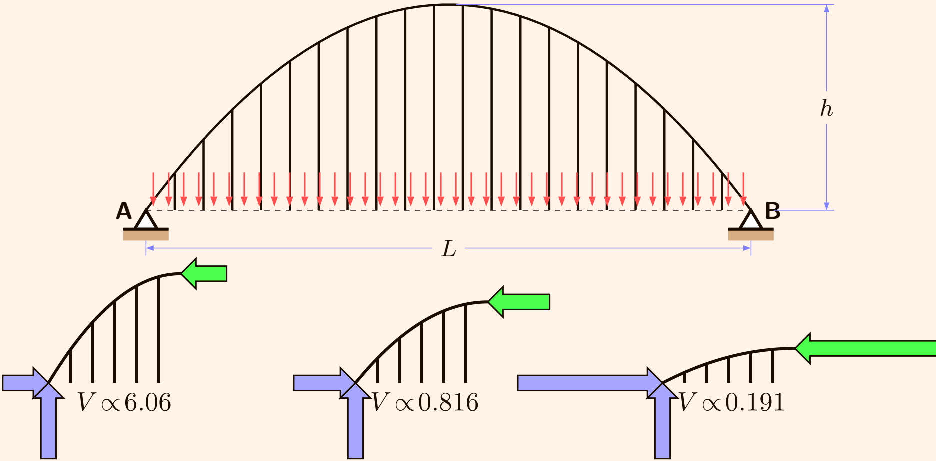


$V_{\mathrm{opt}} \propto 6.06$ if $\mathbf{f}^{+}/\mathbf{f}^{-} = 10$ $V_{\mathrm{opt}} \propto 0.816$ if $\mathbf{f}^{+}/\mathbf{f}^{-} = 1$ $V_{\mathrm{opt}} \propto 0.191$ if $\mathbf{f}^{+}/\mathbf{f}^{-} = 0.1$

This kind of problems was named “fixed boundary” class by Cox (1965), outlining we use displacement constraints (as usual).

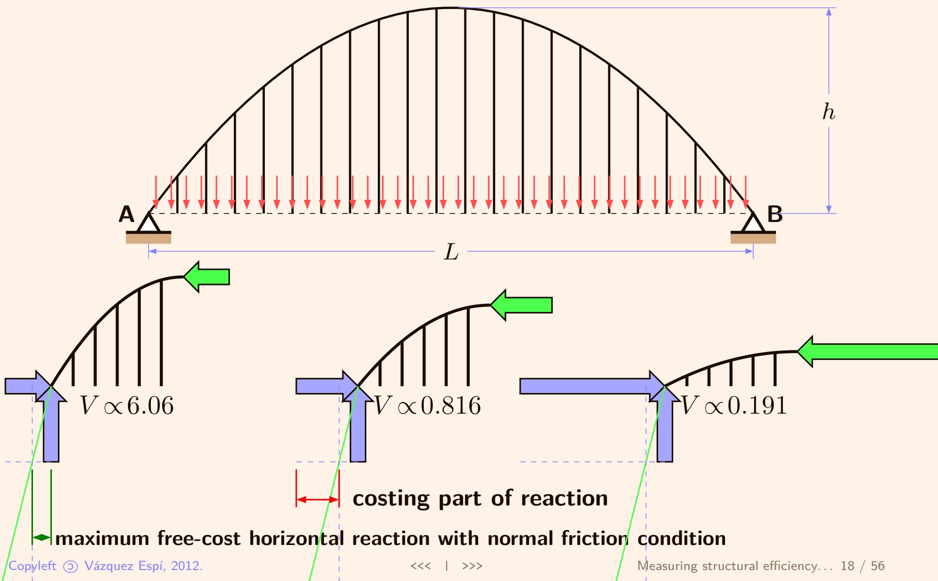
Let us have a look ‘inside’ these solutions. . .

Parabola and Catenary Arches

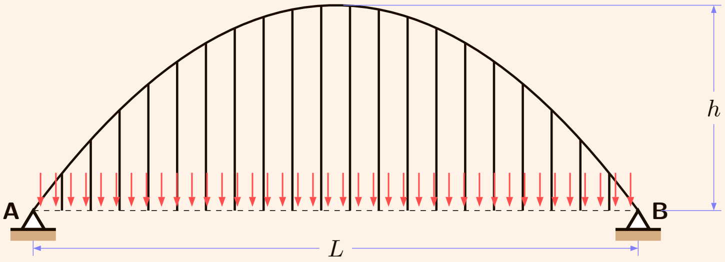


"It must be remembered, nevertheless, that the reactions such as those at [fixed supports], are in any case carried by some other bodies acting as structures and the true picture of the economy achieved should include the abutments." Owen (**1965**:64)

Parabola and Catenary Arches



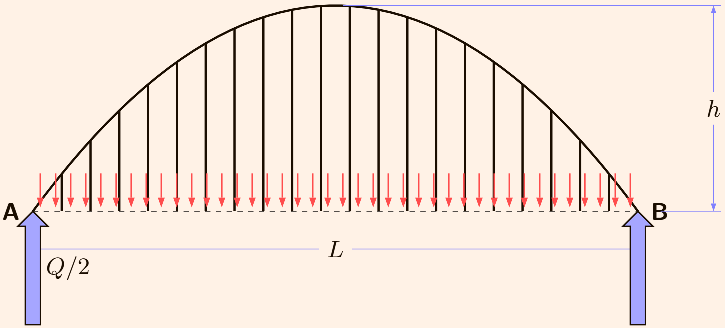
Parabola and Catenary Arches



This kind of structural optimization has a close relation with structural analysis as usual, and **this is an advantage**: to add an optimization module to an analysis program is all we need...

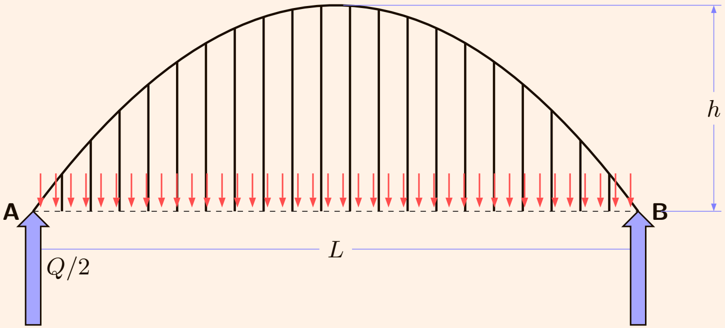
But this is also a **key drawback**: only the cost of analysed structure is accounted, and generally this is lesser than the overall cost of the final design (abutments **A** and **B** are definitely not cost free!).

Parabola and Catenary Arches



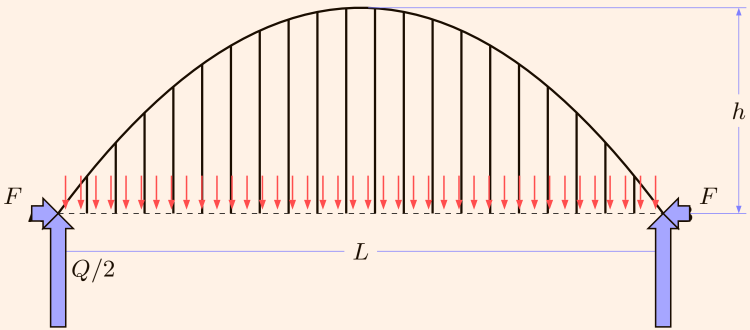
MAXWELL (1870) suggested an alternative approach: to define a set of external forces in equilibrium.

Parabola and Catenary Arches



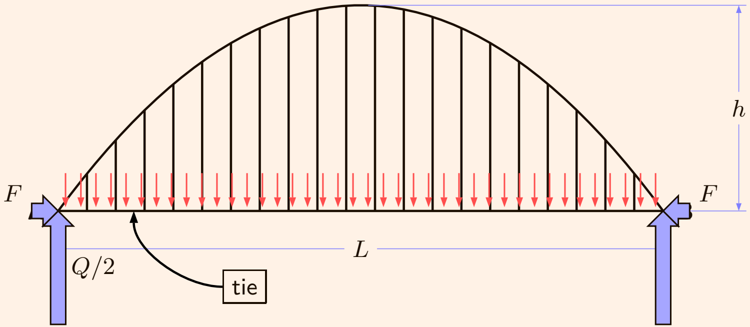
Since vertical reactions are given we can freely decide the horizontal ones (this is COX's "free loading"!). Hence there are infinitely many **Maxwell's problems** that are compatible with our bridge design problem.

Parabola and Catenary Arches



Let us analyse the case in which the horizontal reactions are provided by friction condition between foundation and ground, so the horizontal reaction is a given fraction of the vertical one. Hence, we have **a set of external forces of given magnitude and position in equilibrium** anyway, i.e., a Maxwell problem again.

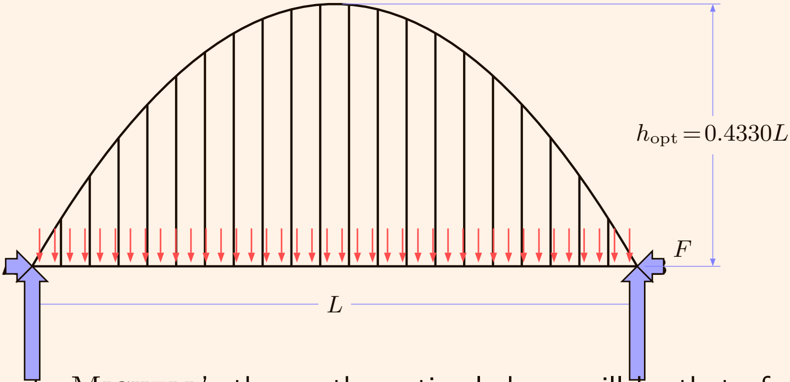
Parabola and Catenary Arches



Since it could be the case that given horizontal reactions will not be enough to equilibrate the arch thrusts, we must add **more** structure, e.g., a horizontal tie between supports.

This completes the definition of a structural scheme that only depends on height h

Parabola and Catenary Arches

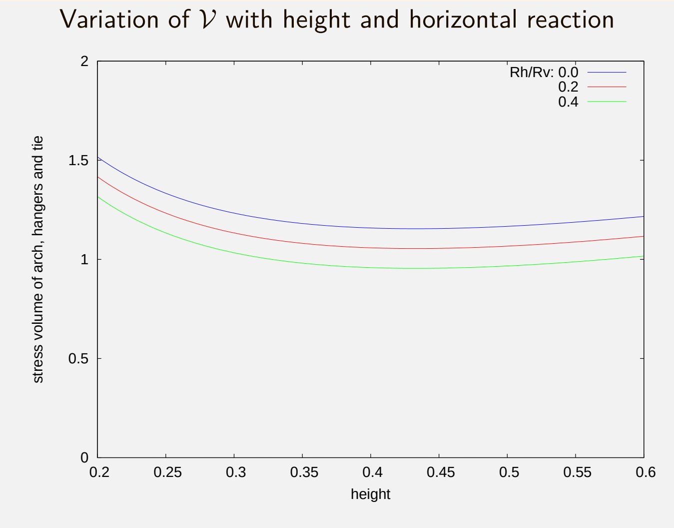


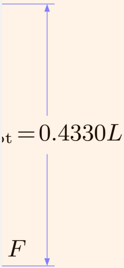
According to MICHELL's theory, the optimal shape will be that of minimum stress volume. Indeed, the geometrical volume will be different for different stress levels (like weight, or other cost), but the **optimal shape will be invariant** as it is the stress volume

$\mathcal{V}_{\rm opt} \propto 1.03$

$\mathbf{f}^+/\mathbf{f}^-$	10	1	1/10
$V_{\rm opt} \propto$	6.23	1.03	0.510

Parabo

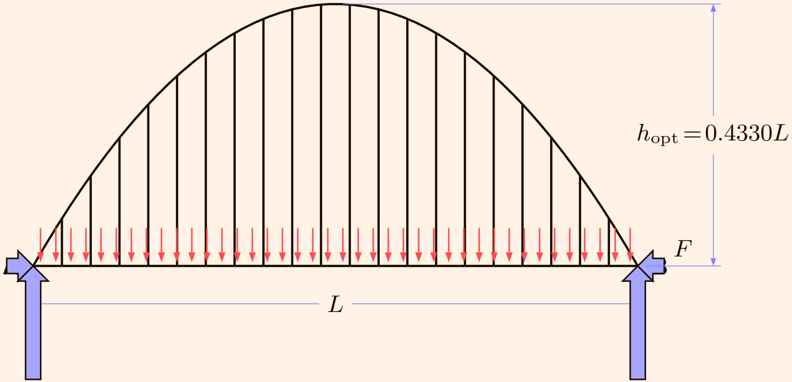




Moreover, the optimal shape is invariant[3] with the horizontal reaction value, although the stress volume is not.

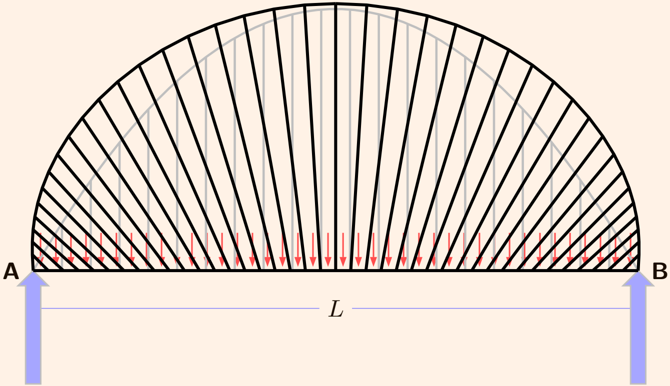
Note also that we have design freedom on h as the curvature of $\mathcal{V}(h)$ is very small near the optimum.

Parabola and Catenary Arches



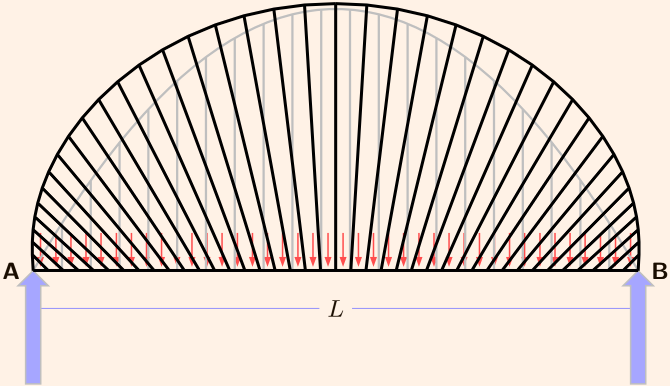
Is this shape the optimum for the bridge design problem?

Parabola and Catenary Arches



This is the best analytical solution we know up to date for $F = 0$, with $\mathcal{V} = 0.985 \mathrm{Q} L$ (Cervera, Vázquez & Vázquez, 2011, submitted to *Engineering Optimization*, 15% lesser than parabolic arch one). We have a better numerical solution, with different shape but similar stress volume (0.97431). Better solutions can exist…

Parabola and Catenary Arches

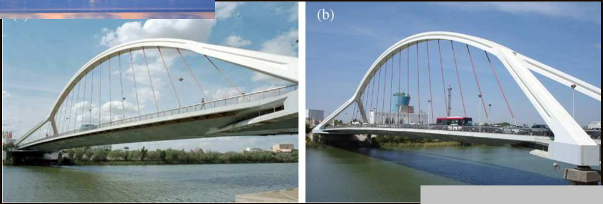


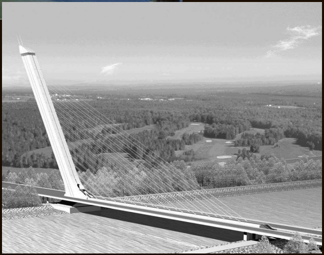
Conclusion: **Within Maxwell & Michell design theory (theory MM), it is possible to measure the structural efficiency of a given sketch through its stress volume.**

Is there a simple measure of structural efficiency with which we can compare different designs for given problems all belonging to the same general family, e.g., "bridges"?

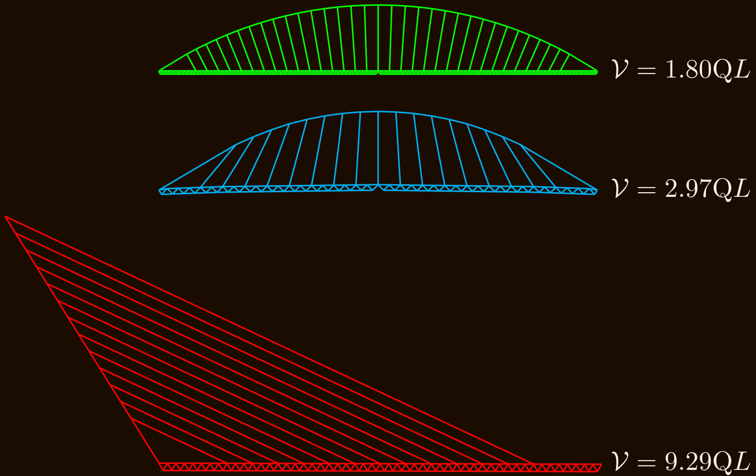
Michell's number: an old scalar in structural design

Different sizes, materials,. . . but the same problem





Three bridges $\rightsquigarrow$ three sketches



Michell's number: an old scalar in structural design

A modest proposition
for measuring structural efficiency
at preliminary stages of design process
(and simultaneously honouring Michell's work)

If the main useful load is Q, and the problem size is L (e.g., bridge span, tower height, etc) then Michell's number μ of a feasible structure under Q-action[4] is defined as:

$$\mu = \frac{\mathcal{V}}{\mathrm{Q}L} \qquad \mathcal{V} = \mu \mathrm{Q}L,$$

hence

the lesser the Michell number,
the greater the structural efficiency.

Michell's number: an old scalar in structural design

We can use a 'dummy' material —$\mathcal{A} = \mathbf{f}/\boldsymbol{\rho}$— for preliminary, trial designs. Then and according to **Aroca's synthesis** on structural design (ca. 1970, theory GMMα), we have:

- **Selfweight:** $\mathrm{P} = \boldsymbol{\rho} V = \boldsymbol{\rho}\dfrac{\mathcal{V}}{\mathbf{f}} = \dfrac{\mathcal{V}}{\mathcal{A}}$ (recall that $\mathcal{A} = \dfrac{\mathbf{f}}{\boldsymbol{\rho}}$)
- **Aroca's hypothesis about useful load and structure's self-weight:**

$$\mu = \frac{\mathcal{V}(\mathrm{Q})}{\mathrm{Q}L} \approx \frac{\mathcal{V}(\mathrm{Q+P})}{(\mathrm{Q+P})L} \approx \frac{\mathcal{V}(\mathrm{P})}{\mathrm{P}\mathcal{L}}$$

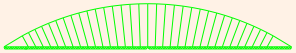
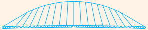
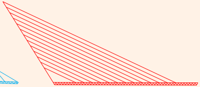
- **Structural scope of a sketch: (Aroca's rule)**

$$\mathcal{V}(\mathrm{P}) \approx \mu \mathrm{P}\mathcal{L} = \mu\frac{\mathcal{V}(\mathrm{P})}{\mathcal{A}}\mathcal{L} \quad \Rightarrow \quad \boxed{\frac{\mathcal{L}}{\mathcal{A}} \approx \frac{1}{\mu}}$$

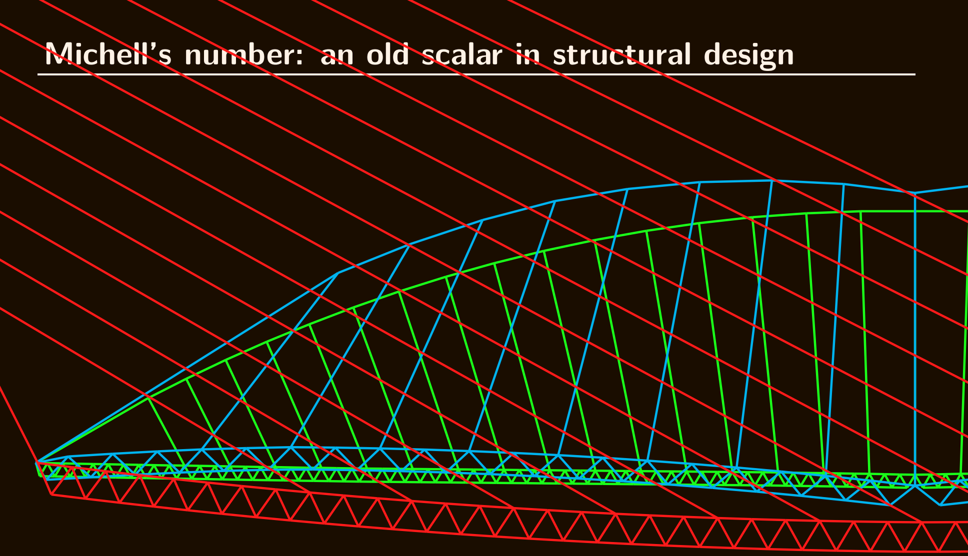
The reciprocal of Michell's number gives us also a rough, useful estimate of sketch's structural scope, relative to material's scope.

(A similar argument can be followed with different materials for tension and compression members.)

Michell's number: an old scalar in structural design

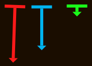
Bridge:	**Apollo**	**La Barqueta**	**Hongshan**[5]
Year:	2005	1989	2005
Sketch:			
Original design:			
Slenderness λ	**3,33**	2,79	1,78
Michell's number $\mathcal{V} \div \mathrm{Q}L$	**1,80**	2,97	9,29
Relative scope $\mathcal{L} \div \mathcal{A}$	***0,557***	*0,336*	*0,107*
Relative size $\frac{1}{10}$ (strength):			
Load cost κ	***1,22***	*1,42*	***15,3***
Selfweight, P/Q	***0,22***	*0,42*	***14,3***
Optimum slenderness design:			
Slenderness λ	**1,20**	1,07	0,469
Michell's number $\mathcal{V} \div \mathrm{Q}L$	**1,14**	1,99	4,58
Relative scope $\mathcal{L} \div \mathcal{A}$	***0,874***	*0,503*	*0,218*
Relative size $\frac{1}{10}$ (strength):			
Load cost κ	***1,13***	*1,25*	***2,62***
Selfweight, P/Q	***0,13***	*0,25*	***1,62***

Michell's number: an old scalar in structural design



Flexibility estimate for fully-stressed sketches:

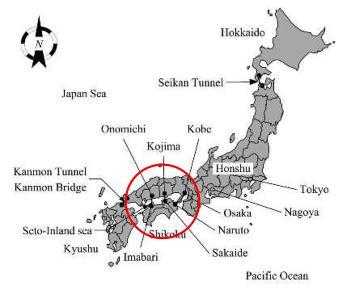
(The lesser the stress volume, the lesser the flexibility)

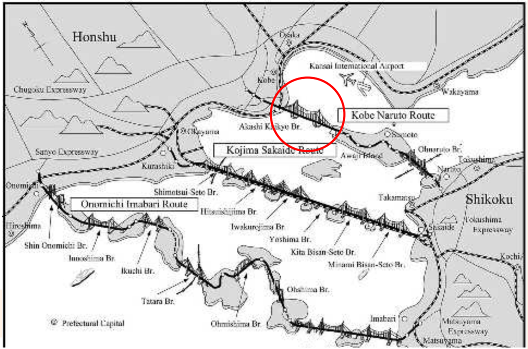


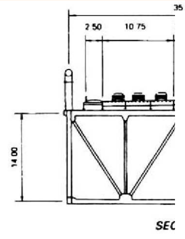
The Akashi Kaikyo Bridge

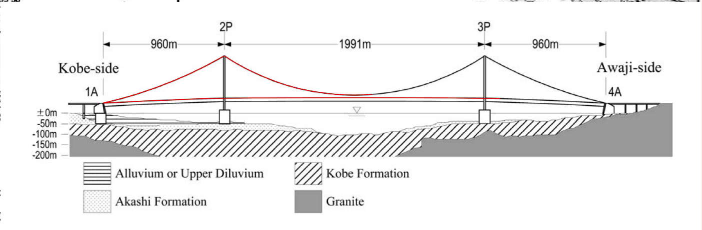
The Akashi Kaikyo bridge was completed in 1998 and has 1.991 km of central span, a slenderness in the order of 6 (span/deep). "The bridge holds three records: it is the longest, tallest, and most expensive suspension bridge ever built." Main fact: very high steels have to be used: one with **allowable stress of $800\,\mathrm{N/mm^2}$ and strength of $1800\,\mathrm{N/mm^2}$ in main cables**. And note that the greater the allowable stress, the greater the strain, so we can hope greater deflection and flexibility, all other things the same.[6].

The Akashi Kaikyo Bridge





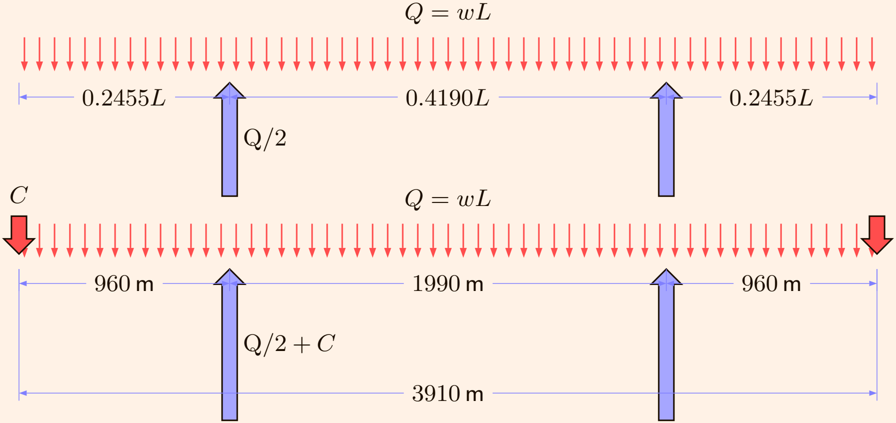




The Akashi Kaikyo Bridge

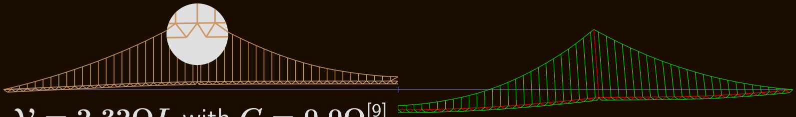
Some Maxwell's problems for estimating AKB's stress volume

$\mathcal{M} = 0.021CL$ [7] **but** $\delta\mathcal{V} \propto \delta\mathcal{C}$ **if** $k^+ = k^-$

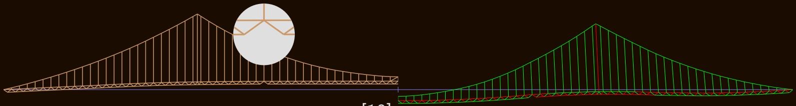


The Akashi Kaikyo Bridge

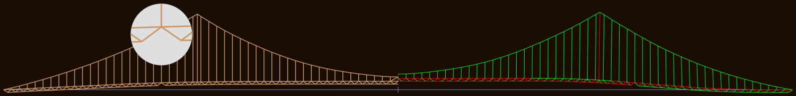
Selecting a C-problem and a Maxwell structure[8]



$\mathcal{V} = 2.32\mathbf{Q}L$ with $C = 0.0\mathbf{Q}$[9]



$\mathcal{V} = 1.81\mathbf{Q}L$ with $C = 0.0\mathbf{Q}$[10]

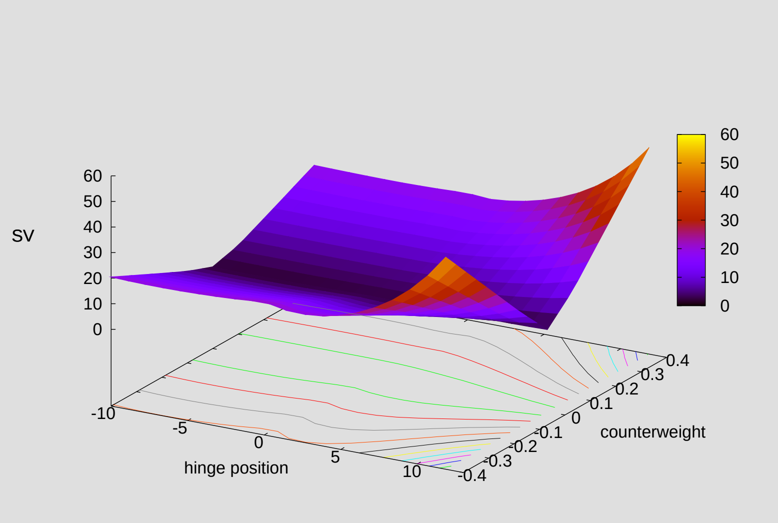


$\mathcal{V} = 1.35\mathbf{Q}L$ with $C = 0.02\mathbf{Q}$[11]

See for details "How to calculate the stress volume from a structure's sketch"

The Akashi Kaikyo Bridge

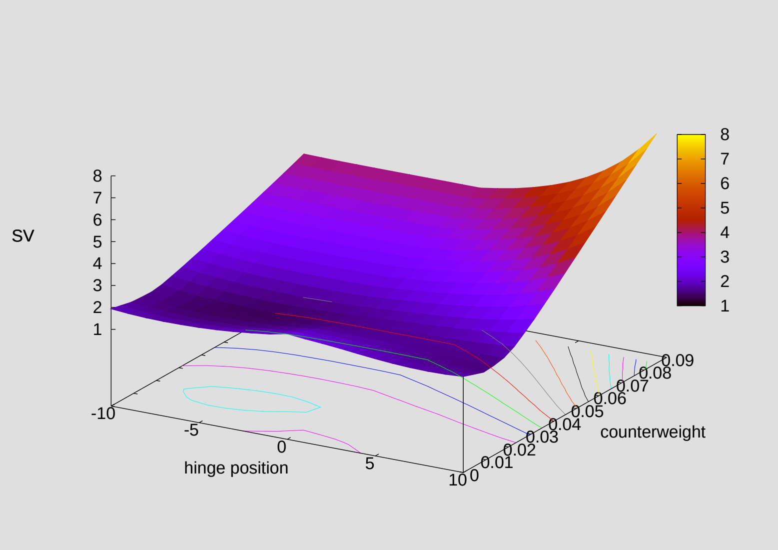
Selecting a C-problem and a Maxwell structure



We have enough design freedom on hinge position but lesser one on C

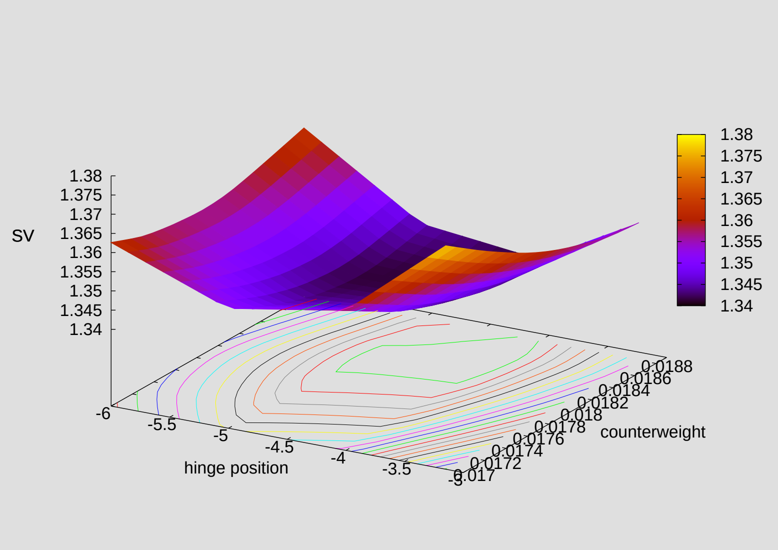
The Akashi Kaikyo Bridge

Selecting a C-problem and a Maxwell structure



The Akashi Kaikyo Bridge

Selecting a C-problem and a Maxwell structure



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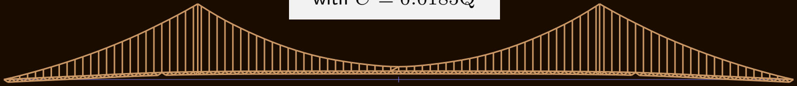
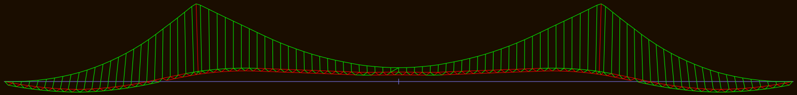
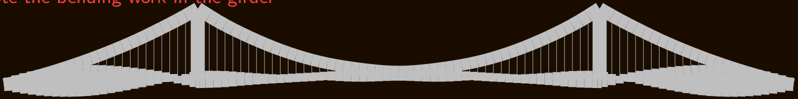
$\mathcal{V} = 1.34\text{Q}L$
with $C = 0.0185\text{Q}$





$\times 4$

Note the bending work in the girder

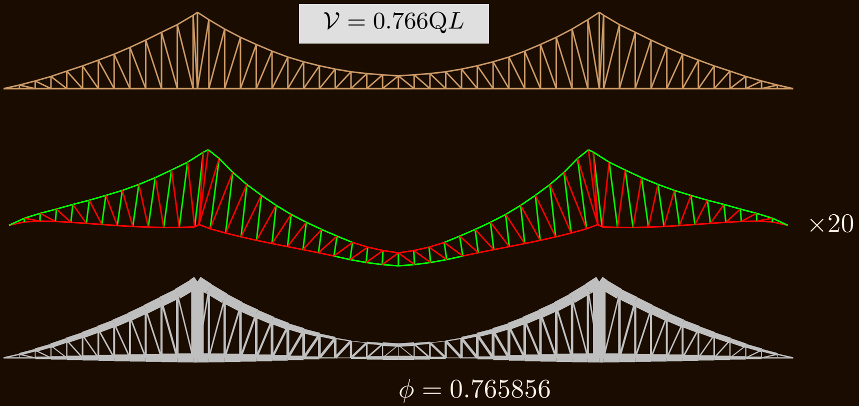


$\phi = 1.34039$

ρ (kN/m^3)	f (MN/m^2)	$\mathcal{A}$ (m)	$\mathcal{L}_{\text{AKB}} \approx$ (m)	$\mathcal{L}_{\text{AKB}} \geq 3910\,\text{m}$	$L/\mathcal{L}$	P/Q$\approx$
78.5	400	5095	3801	false	1.03	∞
78.5	800	10190	7602	true	0.51	1.04
78.5	1800	22928	17105	true	0.23	0.30

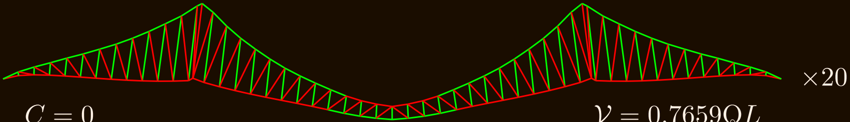
The Akashi Kaikyo Bridge

Better solutions: a simple truss ($C = 0$)[12]

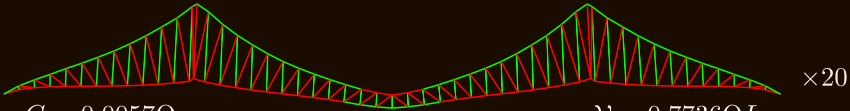


The Akashi Kaikyo Bridge

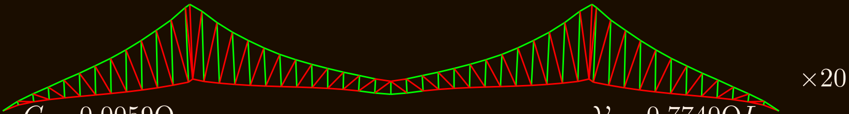
Better solutions: simple trusses with counterweight



$C = 0$ $\mathcal{V} = 0.7659QL$



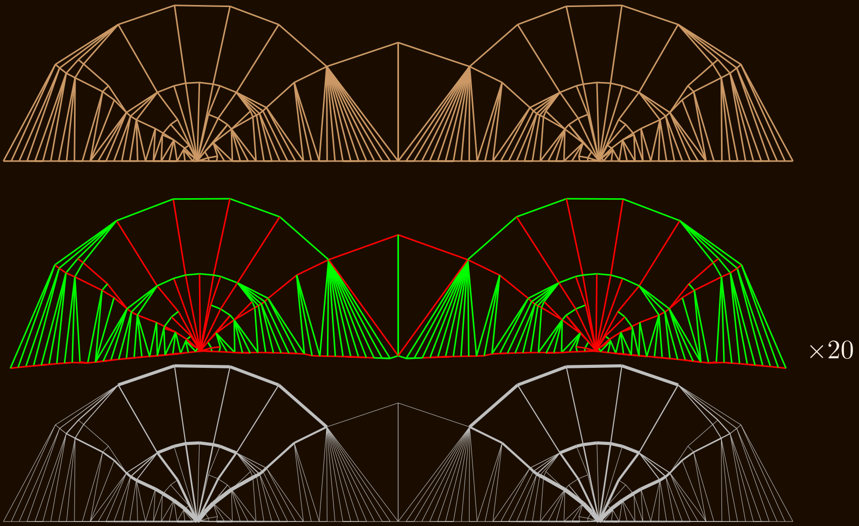
$C = 0.0057Q$ $\mathcal{V} = 0.7736QL$



$C = 0.0059Q$ $\mathcal{V} = 0.7740QL$

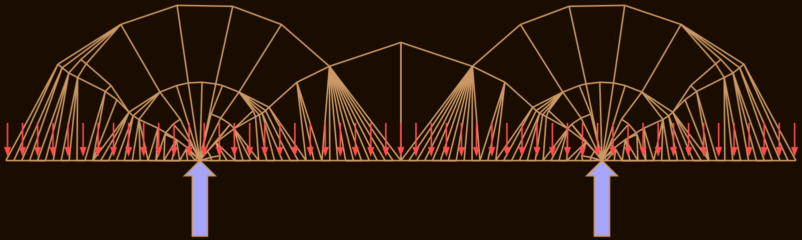
The Akashi Kaikyo Bridge

The best-known one: $\mathcal{V} = 0.4031QL$, $C = 0$, $\mathcal{L}_{\max}/\mathcal{A} > 2.481$, $\chi_{\mathrm{AKB}} = 0.07$

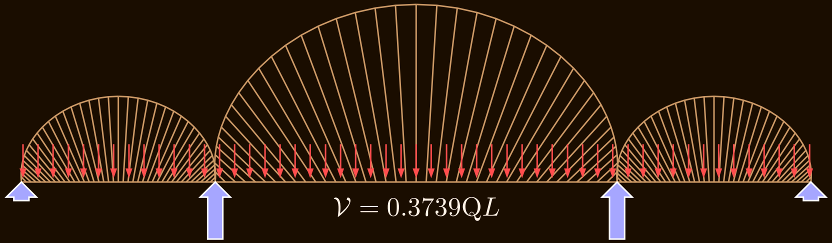


The Akashi Kaikyo Bridge

¿The best-known one?: $\mathcal{V} = 0.4031QL$, $C = 0$, $\mathcal{L}_{\max}/\mathcal{A} > 2.481$, $\chi_{\mathrm{AKB}} = 0.07$



There is a better solution for another Maxwell problem!



$\mathcal{V} = 0.3739QL$

The Akashi Kaikyo Bridge

A provisional conclusion:

The AKB design could have a relative size of 0.15 respect to the best-known solution scope with a steel working at $800\,\mathrm{N/mm^2}$. This would correspond to an efficiency of about 85% (ratio of useful load to total load). But the actual AKB has an efficiency of about 63% which would correspond to a relative size of 0.375, as a consequence of its sub-optimal shape.

Is this a record?

In respect to the absolute scope **for a steel strength of 1800 $\mathrm{N/mm^2}$, the relative size of AKB is about** 0.07. All the remarkable bridges since XIX century seem to have a relative size equal or lesser than 0.10 respect to the material with which they were built. It seems that this figure (10% of the absolute scope on strength) was a practical limit for bridge construction. And it seems that the AKB has not surmounted it.

The Akashi Kaikyo Bridge

And a suggestion:

To improve the allowable stress to enlarge the absolute size of future bridges has a key drawback: to increase their flexibility...

Perhaps, it will be worthy to research how the best-known but up to now theoretical sketches could be built. In this way the absolute size will be increased without loss of flexibility.

(See also El puente Akashi como problema de diseño.)

A look at the past... and the future

On Layout Design and Optimization

	problem class		
date	free load	fixed boundary	fancy classes
1638	Galileo		
1870	Maxwell		
1904	Michell		
Cross (1936) on analysis and design			
Sved (1954) on minimum weight			
1955–1964	Cox[2], Hemp[2]	Best, Chan	
1965–1974	Cox, Owen, Parkes	Hemp[4], Razani	
	Prager[3]	Prager[9]	
	Aroca, de Miguel, Parkes	Rozvany	
	McConnel (1974)		
1975–1984		Rozvany[7]	Rozvany[5]
1985–1994	Cervera[2]	Rozvany[many]	Rozvany[many]
1995–2004	Vázquez *et alii*[4], French[2]	Rozvany[many]	Rozvany[many]
Rozvany (1996) on Michell's error			
2005–	Bouchitté *et alii* (2008)		Darwich *et alii*
	Cervera, Vázquez *et alii*[4]	Rozvany[many]	Rozvany[many]
Vázquez & Cervera (2011, 2012a, 2012b,...) on Rozvany's errors			

A look at the past. . . and the future

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2005–	Bouchitté *et alii* (2008)		Darwich *et alii*
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A look at the past. . . and the future

On Layout Design and

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	AROCA, DE MIGUEL, PARKES	ROZVANY	
	MCCONNEL (1974)		
1975–1984		ROZVANY[7]	ROZVANY[5]
1985–1994	CERVERA[2]	ROZVANY[many]	ROZVANY[many]
1995–2004	VÁZQUEZ *et alii*[4], FRENCH[2]	ROZVANY[many]	ROZVANY[many]
ROZVANY (1996) on Michell's error			
2005–	BOUCHITTÉ *et alii* (2008)		DARWICH *et alii*
	CERVERA, VÁZQUEZ *et alii*[4]	ROZVANY[many]	ROZVANY[many]
VÁZQUEZ & CERVERA (2011, 2012a, 2012b,. . .) on Rozvany's errors			

Interestingly enough, what remains [after abstract reduction] is called "real", i.e., considered as more important than reality.

FEYERABEND (1999): "Conquest of abundance: a tale of abstraction versus the richness of being"

A look at the past... and the future

On Layout Design and Optimization

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A look at the past. . . and the future

On Layout Design and Optimization

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A look at the past. . . and the future

Utinam tam le possem vera reperire, qun falsa convincere

MARCUS TULLIUS CICERO, *De Natura Deorum*, I, 91.

Ojalá fuera tan fácil descubrir la verdad, como desvelar la falsedad — If only it were so easy to discover the truth, as uncover the falseness

1638	GALILEO		
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A look at the past. . . and the future

Open problems of the Maxwell & Michell design theory (free load) and Galileo's structural scope theory.

- Only in few cases, the absolute optima are known. To search for optimum or nearly-optimum layouts —both analytical and numerical ones— gives a lot of opportunities for discoveries. . .
- The original theory (MM) cannot tackle selfweight. Aroca's synthesis shows a way to connect Galileo's theory (G) with the former. This synthesis (GMMα) is exact for isomorphic selfweight and useful load, but generally there is not such isomorphic relationship. To extent this synthesis towards a GMMβ theory is an interesting theoretical challenge.
- Optimality criteria for solutions of null efficient and maximum size are not known (G theory). The classic constant stress shapes are not actually feasible solutions because their stress tensors are not bounded neither constant. This is another theoretical challenge.

Measuring structural efficiency
in
bridges' sketches
or
Lo que cunde un año en CIMNE Barcelona

Mariano Vázquez Espí

GIAU+S (UPM)
Grupo de Investigación en Arquitectura, Urbanismo y Sostenibilidad
Universidad Politécnica de Madrid
http://habitat.aq.upm.es/gi

Edición del 22 de febrero de 2012
Compuesto con *free software*:
GNULinux/LaTeX/dvips/ps2pdf

Funicular curve theory

For any funicular curve $y = f(x)$ and vertical weights:

$$e(x) = H\sqrt{1 + {y'}^2} \qquad H \text{ being the horizontal reaction} \tag{1}$$

For constant cross-section A with selfweight only — the catenary case:

$$V = A \int_{\mathrm{A}}^{\mathrm{B}} \mathrm{d}s = 2A \int_0^{L/2} \sqrt{1 + {y'}^2}\,\mathrm{d}x \qquad A \text{ being } \frac{|e|_{\max}}{\mathsf{f}} \tag{2}$$

For strict sizing (variable A) — the general case:

$$V = \int_{\mathrm{A}}^{\mathrm{B}} A\,\mathrm{d}s = 2\frac{H}{\mathsf{f}} \int_0^{L/2} 1 + {y'}^2\,\mathrm{d}x \tag{3}$$

<<<

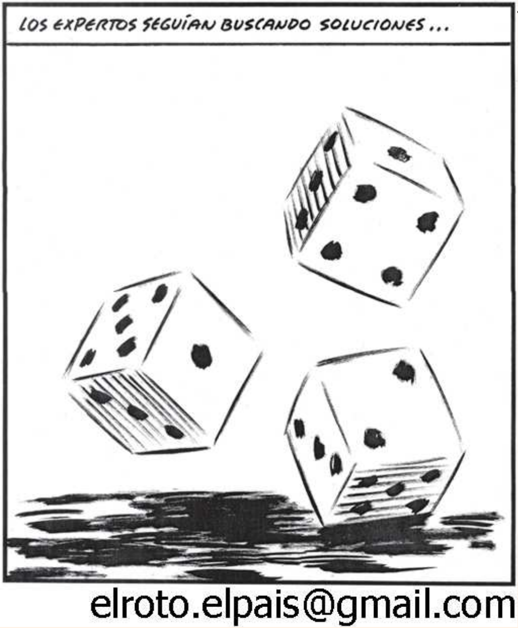
Physical cost after classic thermodynamics

Clausius (1985) wrote a booklet of 26 pages intitled *Über die Energievorräte der Natur und ihre Verwertung zum Nutzen der Menschheit*.
He pointed out several important remarks on economics, among them:

- **The Second Law always holds: there is no escape.**
- We must account for **the physical cost of all we need to reach a given target**, being this **the fundamental axiom of thermodynamics on accounting**.

  In respect to mankind business, we cannot simply suppose that coal, iron, nitrogen compounds and other materials we found in Nature are cost-free.[13] There is no free-cost resources in the Earth but Sun's exergy. (Not surprisingly, this is a very, very old idea.)
- If standard economics is not able of accounting the cost of the so named free resources (especially mineral ones, earth surface, water, etc), then standard calculus of the best way **towards a welfare state** will lead us actually **towards a random state** in spite of all claimed good intentions of economists.[14]

Physical cost after classic thermodynamics



Physical cost after classic thermodynamics

MAXWELL —well advised of thermodynamical matters— stated the structural design problem in this very thermodynamical realm —and not surprisingly— in the form of only three minor annotations in his well known paper "On reciprocal figures, frames and diagrams of forces" of 1870.

Physical cost after classic thermodynamics

MAXWELL —well advised of thermodynamical matters— stated the structural design problem in this very thermodynamical realm —and not surprisingly— in the form of only three minor annotations in his well known paper “On reciprocal figures, frames and diagrams of forces” of 1870.

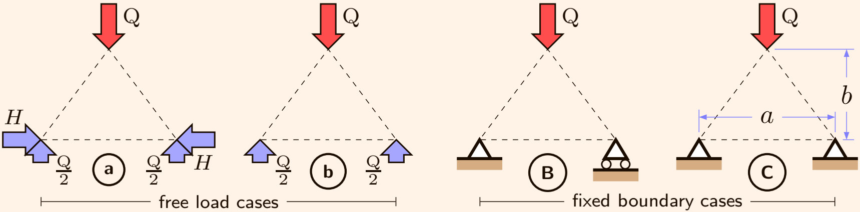
MAXWELL works **only** with a especial kind of problems, those of **external forces —of given magnitude and position— in equilibrium**. In this manner, the fundamental axiom on accountability holds, since there is not exergy transfer across system’s boundary.

Physical cost after classic thermodynamics

MAXWELL works **only** with a especial kind of problems, those of **external forces —of given magnitude and position— in equilibrium**.

They were named "free load" class (COX, 1965) or "Maxwell's problems" (CERVERA, 1989).

As **this fact** is the fundamental axiom of the design theory, let us consider some examples for given useful load Q and geometry a, b:



b and **B** are equivalent but **a** and **C** are not. In fact, the intersection of the two classes is the subset of problems with statically determinate support condition.[15]

Physical cost after classic thermodynamics

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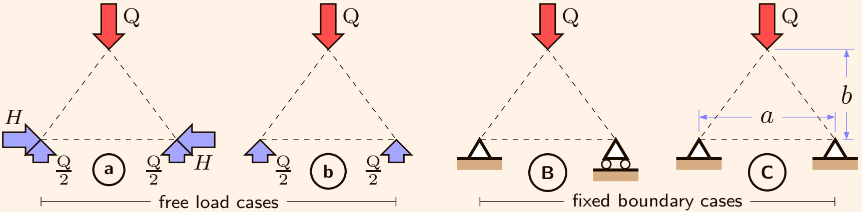

We consider an infinite set of solutions into the free load class (on the left) and we look for a good feasible one as a guide for a new design. Besides, we consider a given solution into the fixed boundary class (on the right) and we analyse it to realise its performance; in this class, the selected supports have the role of other existing structures and we suppose they can sustain the analysed one.

Physical cost after classic thermodynamics

MAXWELL works **only** with a especial kind of problems, those of **external forces —of given magnitude and position— in equilibrium**.

The kernel of the theory has three main elements:

Maxwell's lemma

Maxwell's conjecture

Michell's lemma

Physical cost after classic thermodynamics

MAXWELL works **only** with a especial kind of problems, those of **external forces —of given magnitude and position— in equilibrium**.

Maxwell's lemma.

> For all structures that solve a Maxwell problem, the Maxwell number $\mathcal{M}$ is invariant:
>
> $$\mathcal{M} = \int_V \sigma \, \mathrm{d}V = \sum_i e_i \ell_i$$
>
> where e is the internal force in each member, being ℓ its length; V stands for all the geometric volume of the structure. (Proof: apply virtual work principle with an unitary expansion, $\varepsilon = 1$)

Maxwell's conjecture

Michell's lemma

Physical cost after classic thermodynamics

MAXWELL works **only** with a especial kind of problems, those of **external forces —of given magnitude and position— in equilibrium**.

Maxwell's lemma: $\mathcal{M} = \int_V \sigma \, \mathrm{d}V \; = \sum_i e_i \ell_i$ and $\delta\mathcal{M} = 0$

Maxwell's conjecture. The **total** quantity of material needed for solving a structural design problem with a given structure that solves a compatible Maxwell problem would be proportional to:

$$\mathcal{V} = \int_V \mathsf{abs}\,(\sigma)\; \mathrm{d}V \; = \sum_i \mathsf{abs}\,(e_i)\,\ell_i$$

This "quantity" (MICHELL, 1904) is simply the **stress volume** of the structure. As the integral operator is a lineal one, we can write:

$$\mathcal{V} = \mathcal{V}^+ + \mathcal{V}^- \qquad\qquad \mathcal{M} = \mathcal{V}^+ - \mathcal{V}^-$$

i.e., we can decompose the integrals in traction and compression parts, or indeed any other parts we would wish (e.g., horizontal and vertical parts, etc).

Michell's lemma

Physical cost after classic thermodynamics

MAXWELL works **only** with a especial kind of problems, those of **external forces —of given magnitude and position— in equilibrium**.

Maxwell's lemma: $\mathcal{M} = \int_V \sigma \, \mathrm{d}V = \sum_i e_i \ell_i$ and $\delta\mathcal{M} = 0$. $\mathcal{M} = \mathcal{V}^+ - \mathcal{V}^-$

Maxwell's conjecture: total cost $\propto \mathcal{V} = \int_V \mathsf{abs}\,(\sigma)\, \mathrm{d}V = \sum_i \mathsf{abs}\,(e_i)\,\ell_i$ when $\delta\mathcal{M} = 0$. $\mathcal{V} = \mathcal{V}^+ + \mathcal{V}^-$

Corollary. Since $\mathcal{M} = \mathcal{V}^+ - \mathcal{V}^-$, if $\delta\mathcal{M} = 0$ then $\delta\mathcal{V}^+ = \delta\mathcal{V}^-$.

Michell's lemma

Physical cost after classic thermodynamics

MAXWELL works **only** with a especial kind of problems, those of **external forces —of given magnitude and position— in equilibrium**.

Maxwell's lemma: $\mathcal{M} = \int_V \sigma \, \mathrm{d}V = \sum_i e_i \ell_i$ and $\delta\mathcal{M} = 0$. $\mathcal{M} = \mathcal{V}^+ - \mathcal{V}^-$

Maxwell's conjecture: total cost $\propto \mathcal{V} = \int_V \mathsf{abs}(\sigma) \, \mathrm{d}V = \sum_i \mathsf{abs}(e_i) \, \ell_i$ when $\delta\mathcal{M} = 0$. $\mathcal{V} = \mathcal{V}^+ + \mathcal{V}^-$

Corollary. Since $\mathcal{M} = \mathcal{V}^+ - \mathcal{V}^-$, if $\delta\mathcal{M} = 0$ then $\delta\mathcal{V}^+ = \delta\mathcal{V}^-$.

These annotations of MAXWELL were generally ignored during years—as CLAUSIUS's remarks on economics were— until MICHELL (1904) realised their importance in perhaps the most cited paper on truss-layout optimization: "The Limits of Economy of Material in Frame-structures" (only 9 pages!). He adopted a similar approach that GIBB's in statistical thermodynamics for assigning "the forms of frames which attain the limit of economy". With this aim, he firstly proved Maxwell's conjecture.

Michell's lemma

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Michell's lemma. For any cost $\mathcal{C}$ defined as $\mathcal{C} = k_+\mathcal{V}^+ + k_-\mathcal{V}^-$ with $k_+ \geq 0$, $k_- \geq 0$ and $k_+ + k_- > 0$, if $\delta\mathcal{M} = 0$ then the following two problems are equivalent:

$$\min \mathcal{C} \quad \Leftrightarrow \quad \min \mathcal{V}$$

Proof: Consider the variation of:

$$\mathcal{C} = \frac{1}{2}\left\{(k_+ + k_-)\cdot\mathcal{V} \; + \; (k_+ - k_-)\cdot\mathcal{M}\right\}$$

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Michell's achievement was ignored until ca. 1945. After the books of COX (1965), OWEN (1965) and PARKES (1965), the design theory of MAXWELL became a standard—as it was the case of CLAUSIUS's remarks after books like "The entropy law and the economic process" by GEORGESCU-ROEGEN (1971). Unfortunately, many people —scientists, technicians, decision makers, politicians,. . . — continued using **partial cost accounting** in both cases and hence finding at best **random solutions** while they were looking for optimum ones. Worst: the number of these persons had been increased many times until now.

Physical cost after classic thermodynamics

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Anyway, for fully-stressed arches we have:

$$V = \frac{\mathcal{V}^+}{\mathsf{f}_+} + \frac{\mathcal{V}^-}{\mathsf{f}_-}$$

How to calculate the stress volume from a structure's sketch

A design problem is defined by the useful load Q and some constraints on the geometry of feasible solutions. The latter can be of very different nature, e.g., in the AKB case they determine the support position and require solutions placed over the load line.

There can be infinitely many Maxwell's problems that include the useful load and fulfill the considered constraints. To select a Maxwell problem we must give —fully defined— a set of external forces in equilibrium.

A Maxwell problem had infinitely many feasible solutions. A typical question is what will be the optimal solution.

How to calculate the stress volume from a structure's sketch

If we wish to estimate the stress volume of a built structure, we must make the following:

- To define the design problem with all conditions we need so as its feasible solutions will represent a sketch of the built structure.
- To find a feasible solution for a Maxwell problem (possibly among infinitely many ones) that it will lead to the structure's sketch with minimal stress volume

Of course the built structure can perform worst than what the selected solution could suggest. But the key point is that the built structure could not perform better.

How to calculate the stress volume from a structure's sketch

In the AKB example, the selected set of Maxwell's problems is described with counterweight C (there are more Maxwell's problems indeed). The set of feasible solutions is described with the position of the girder hinge (the hinge could be in the bottom side of the girder as well), let it be P, counting to the right from the left support ($P = 0$). So we must solve

$$\min_{C,P} \mathcal{V}$$

to select the Maxwell problem and solution which will have the role of the built AKB for our calculus of $\mathcal{V}$. We are implicitly supposing that the counterweight C is free of cost (that is no real in anyway).

How to calculate the stress volume from a structure's sketch

To compute $\mathcal{V}$ for each feasible solution, we can use graphical methods (as Maxwell did).

Nowadays we can better use a standard, structural analysis code. But note the following: as we have a set of external forces in equilibrium, we only need statically determinate support condition for the code works: in fact, only three conditions to eliminate rigid-body motion in 2D case. And we can select these conditions freely (this "free boundary" is the counterpart of the COX's "free load"): if we fix a point and fix the direction of a line across it we are defining these point and line as the reference of measured **virtual** displacements and rotations.

A standard code will ask you for cross-sectional areas, Young modulus of material and so on: you can answer that you like, since the *analysis case* is of statically determinate support condition all these data have no influence on stress results. In this manner, we get the internal forces of the solution (independently of the selected support conditions), and also a rather arbitrary **virtual** displacements of this solution measured respect **the selected reference**.

How to calculate the stress volume from a structure's sketch

There is a third way: to follow the Maxwell's method and to solve the equilibrium equations with internal and external forces. As many internal forces so many equations we will have. We can get also virtual displacements with standard code using cross-sectional areas equal to absolute value of calculated internal forces, but 1 when the latter is null. We must define again an appropriate reference. These virtual displacements can be viewed as the corresponding ones to an absolute upper bound of the real strains. But it is to be noted also that these virtual displacements can be turned real by appropriate preloading [see Sved, 1954], i.e., with generally small changes of the sketched shape.

In the AKB example the third way was used, and the edges of L define the reference line and its mid point the reference point. And of course, the resulting deformed shapes **are not real**: for getting a real deformation we would have to specify a structural material and cross-section of all members... But recall that up to now we have **only** internal stresses in equilibrium.

As the "free boundary" approach can be striking (as opposite to the usual "fixed boundary" approach), it will be worthy to say a bit more about it.

How to calculate the stress volume from a structure's sketch

Consider a number N of 2D-points and E bars joining pairs of them, a finite truss in general sense. The **complete set** of equilibrium equations are:

$$\boldsymbol{f} = {}^{C}\boldsymbol{H}\boldsymbol{e}$$

$\boldsymbol{f}$ being the $2N$ force components, and ${}^{C}\boldsymbol{H}$ an $2N \times E$ matrix. MAXWELL proved that $E \geq 2N-3$ must hold for feasible trusses in equilibrium, i.e., trusses that can not be deformed without external work. Furthermore SVED (1954) proved that the minimal structure has no redundant members for any given $\boldsymbol{f}$.

How to calculate the stress volume from a structure's sketch

If $\boldsymbol{f}$ is a set of forces **in equilibrium**, three equations must be lineal combination of the others, hence ${}^{C}\boldsymbol{H}$ has at most rank $2N-3$.

The **complete set** of kinematic equations (for small displacements) are:

$$\boldsymbol{\Delta} = {}^{C}\boldsymbol{B}\boldsymbol{d}$$

where $\boldsymbol{d}$ is the displacement vector, $\boldsymbol{\Delta}$ the elongation vector, and ${}^{C}\boldsymbol{B}$ the transpose of ${}^{C}\boldsymbol{H}$, ${}^{C}\boldsymbol{B} = {}^{C}\boldsymbol{H}'$. As the ranks of ${}^{C}\boldsymbol{B}$ and ${}^{C}\boldsymbol{H}$ are equal, at least three components of $\boldsymbol{d}$ can be chosen freely.

For an SVED's minimal truss with a dummy material, we can choose $\boldsymbol{\Delta} = \mathrm{sgn}(\boldsymbol{e})\ell$ —being $\boldsymbol{e}$ determined by Maxwell's method— and select freely three displacement components as the fixed ones. With ${}^{C}\boldsymbol{B} = [\boldsymbol{B}_\star \quad \boldsymbol{B_3}]$, and $\boldsymbol{d}' = \left\{\boldsymbol{d}_\star{}' \quad \boldsymbol{d_3}'\right\}$:

$$\boldsymbol{d}_\star = \boldsymbol{B}_\star{}^{-1}\left\{\mathrm{sgn}(\boldsymbol{e})\ell - \boldsymbol{B_3}\boldsymbol{d_3}\right\}$$

i.e., a Maxwell solution ($\boldsymbol{e}$) has infinitely many compatible virtual displacement sets, each of them depending on our selection of $\boldsymbol{d_3}$. (This is the virtual work principle again.)

How to calculate the stress volume from a structure's sketch

Once the minimization problem has been solved, we have the value of $\mathcal{V}$ with which we characterize the sketch, e.g., $\mathcal{V}_{\mathrm{AKB}}$.

(Back to the AKB section)

El puente Akashi como problema de diseño

El cálculo del volumen de tensiones del puente Akashi se realizó con **la información disponible** —véanse las referencias—, pero ésta es siempre insuficiente. En particular la geometría del cable principal sigue la forma publicada (lámina 37), y esta forma es una **variable crucial**.

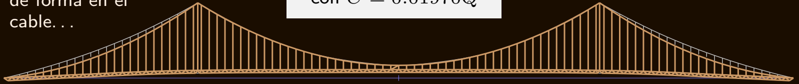
Puede adoptarse un enfoque distinto y considerar el diseño *ex novo* del puente adoptando la misma definición geométrica en cuanto a luces, altura de las pilastras y forma del tablero, pero trazando de nuevo la forma del cable principal como la funicular para una carga uniforme, es decir, parábolas. Al operar así no se tiene en cuenta la influencia en la forma del peso del propio cable, la parte de "catenaria" correspondiente. La ventaja es que ahora todo el conjunto de fuerzas para el equilibrio de la carga útil está determinado sin ambigüedad una vez se defina un problema de Maxwell.

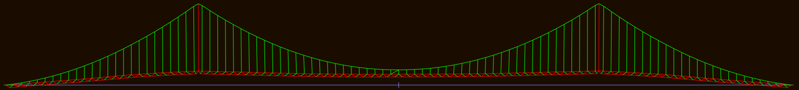
Conservando la decisión de que la carga útil se equilibra mediante reacciones verticales en los apoyos interiores, las necesidades de anclaje en los extremos del cable principal son una fuerza horizontal de $0.41\mathrm{Q}$ y una vertical de $0.036\mathrm{Q}$. Si se adopta el modelo de auto-anclaje como en el análisis precedente, mediante la compresión del tablero, simplemente se requiere un contrapeso de $0.020\mathrm{Q}$ en los extremos.

El puente Akashi como problema de diseño

$$\mathcal{V} = 1.006\mathrm{Q}L$$
con $C = 0.01970\mathrm{Q}$

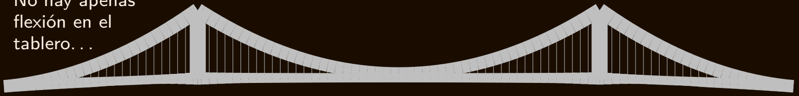
Nótese la diferencia de forma en el cable. . .





$\times 4$

No hay apenas flexión en el tablero. . .



$$\phi = 1.00452$$

$\boldsymbol{\rho}$ (kN/m^3)	f (MN/m^2)	$\mathcal{A}$ (m)	$\mathcal{L}_{\mathrm{AKB}} \approx$ (m)	$\mathcal{L}_{\mathrm{AKB}} \geq 3910\,\mathrm{m}$	$L/\mathcal{L}$	$\mathrm{P/Q} \approx$
78.5	400	5095	5066	true	0.77	3.35
78.5	800	10190	10131	true	0.39	0.64
78.5	1800	22928	22796	true	0.17	0.20

El puente Akashi como problema de diseño

¿Qué volumen de tensiones supone el auto-anclaje? Basta calcular el volumen de tensiones cuando se añaden como fuerzas externas las necesarias para anclar por gravedad el cable principal, y calcular la diferencia: el auto-anclaje mediante la compresión del propio tablero supone $0.40\mathrm{Q}L$, siendo el volumen de la estructura anclada de tan sólo $0.60\mathrm{Q}L$. Puesto que la carga total es del orden de 2252 MN y la longitud total 3910 m, los volumenes de tensiones son $3520\mathrm{GJ}$ y $5280\mathrm{GJ}$, respectivamente.

La fuerza horizontal de anclaje para esa carga ronda los 916 MN (concordante con los datos disponibles, véase la nota [6]), lo que exige un peso en el punto de anclaje de 3136 MN para un coeficiente de rozamiento de 0.3 (igualmente concordante).

Por otra parte, como la razón entre peso propio de la estructura anclada y la carga útil para el acero empleado es de 0.30, la eficiencia en carga previsible sería de $1/1.30$, es decir, del $77\,\%$, que se compara bien con la estimación disponible del $63\,\%$ para el diseño real.

El puente Akashi como problema de diseño

Nótese finalmente que la proporción entre volumen de tensión y peso real es de 561 m para el anclaje por gravedad y de 3750 m para la estructura anclada.

La discordancia entre ambos alcances muestra el error que se comete al representar el anclaje externo mediante el auto-anclaje. Una contabilidad ajustada sólo puede hacerse para cada coste $\mathcal{C}$ de interés (peso, emisiones contaminantes, etc) y una vez fijados los costes por unidad de volumen de tensión para el material estructural y por unidad de peso para el anclaje.

Sin embargo, en el diseño preliminar —sin apenas datos de cómo serán las cosas al final— es muy práctico representar el coste de elementos “externos” —como el anclaje por gravedad— mediante el coste de la estructura necesaria para realizar “internamente” sus funciones. De este modo, al menos **todos** los costes son contabilizados aunque sea de forma abstracta.

En todo caso recuérdese que, a la hora de comparar diseños alternativos, el coste de tales elementos externos, tanto si son necesarios como si no, puede incluirse o excluirse a voluntad según se defina el conjunto de fuerzas externas en equilibrio utilizado para evaluar cada solución considerada.

<<<

Layout scope, general formulation

Note on different materials in tension and compression.

Recall that $\mathcal{V}=\mathcal{V}^{+}+\mathcal{V}_{-}$ and $\mathcal{M}=\mathcal{V}^{+}-\mathcal{V}^{-}$. According to **Aroca's synthesis** on structural design (ca. 1970), we have:

- **Selfweight:** $\mathrm{P}=\boldsymbol{\rho}V=\boldsymbol{\rho}^{+}\dfrac{\mathcal{V}^{+}}{\mathsf{f}^{+}}+\boldsymbol{\rho}^{-}\dfrac{\mathcal{V}^{-}}{\mathsf{f}^{-}}=\dfrac{\mathcal{V}^{+}}{\mathcal{A}^{+}}+\dfrac{\mathcal{V}^{-}}{\mathcal{A}^{-}}$
- **Aroca's hypothesis about useful load and structure's self-weight for a size L:**

$$\frac{\mathcal{V}(\mathrm{Q})}{\mathrm{Q}}\approx\frac{\mathcal{V}(\mathrm{Q}+\mathrm{P})}{\mathrm{Q}+\mathrm{P}}\approx\frac{\mathcal{V}(\mathrm{P})}{\mathrm{P}}$$

- **Structural scope of a sketch: (Aroca's rule)**

$$\mathcal{V}\big|_{L=\mathcal{L}}\approx\mu\mathrm{P}\mathcal{L}=\frac{1}{2}\mu\mathcal{L}\left(\frac{\mathcal{V}+\mathcal{M}}{\mathcal{A}^{+}}+\frac{\mathcal{V}-\mathcal{M}}{\mathcal{A}^{-}}\right)\quad\Rightarrow$$

$$\Rightarrow\quad\boxed{\mathcal{L}\approx\frac{\mathcal{A}^{+}}{\mu}\frac{2}{1+\dfrac{\mathcal{A}^{+}}{\mathcal{A}^{-}}+\left(1-\dfrac{\mathcal{A}^{+}}{\mathcal{A}^{-}}\right)\dfrac{\mathcal{M}}{\mathcal{V}}}}$$

Credits

- co-workers: Joaquín Antuña Hevia, Antonio Aznar López, Jaime Cervera Bravo, Carlos Olmedo Rojas, Jesús Ortiz Herrera, Carlos Vázquez Espí
- advisers, discussers and reviewers: Tino Bértolo, Fernando Cembranos, María Cifuentes, Blanca Estevan, Lucía Gutiérrez Vázquez, Belen Gopegui, José Ignacio Hernando, Victor Lindberg, Mont Marsá, Elena Moreno, José Manuel Naredo, Esteban Pujals, Cata Serra, Teresa Ticó, Marta Vázquez Álvarez, Pilar Vázquez (and of course the CIMNE Barcelona crew)
- special thanks for occasional but indispensable help at Barcelona to Alberto Burgos, Carlos Labra, Joaquin Lozano and Rosa María Olea
- retrieving document support: Library of ETS de Arquitectura de Madrid
- hardware: `xfire.cimne.upc.edu` and `vega.cimne.upc.edu` at Barcelona, `hyakutake.ee.upm.es` at Madrid
- software: GNU/Linux, TEX, LATEX, `perl`, `maxima`
- catering: Alimentación Pons, Bar Versalles y "el café de la esquina (sin nombre) que da ensaladas" near the former (Barcelona)
- Vázquez's hosting at Barcelona: Eugenio Oñate (working place) and Teresa Ticó (living place)

To my father, a practical drawer-designer of aircrafts, lorries and many other useful objects.

Notes

[1]This edition is very, very improved as a result of the discussion following the CIMNE Coffee conference. The author thanks to all attendants for their valuable comments, and he hopes that all the questions from Abel Coll, Riccardo Rossi and others (I know the faces, not the names) have now clear answers in the sequel. In a precise sense, the CIMNE Coffees have a very similar nature than the meetings organised by the Royal Society and other scientific association in the XIX century, in which MAXWELL participated. These meetings are (and were) a real peer-review method... <<<

[2] Como veremos, comparar a Calatrava con Gaudí o Candela, cuando se hace en la misma página en la que se reproduce un boceto del puente del Alamillo, es un insulto para estos últimos, sobre todo porque habiendo ambos fallecido, no pueden salir en su propia defensa. <<<

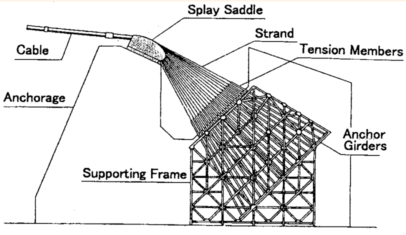
[3] In this case the shape is absolutely invariant. But it is to be stressed that this property has not been fully investigated yet. <<<

[4] Each load hypothesis leads to a different Michell's number. If all of these numbers have been calculated with the same basis $(\mathrm{Q}L)$, the greatest one points to the worst hypothesis. <<<

[5] ¿Por qué he utilizado el puente Hongshan en China, y no el original, El Alamillo en España? Para enfatizar un misterio que me atormentó durante algunas semanas. Para cuando se empezó a proyectar el puente Hongshan ya se habían publicado varios *papers* describiendo los graves problemas de flexibilidad de El Alamillo. Incluso éste, había sido rehabilitado hasta en tres ocasiones a fin de resolver sus problemas de funcionamiento. Teniendo toda esta información

publicada en medios indexados —JCR y similar, que se supone tienen una amplia difusión, cf. Vázquez Espí [2011c]—, ¿cómo explicar que los diseñadores de Hongshan se decidieran por un boceto tan malo y tan feo? Fue el profesor Pujals (de la Universidad Complutense de Madrid) quien me dio la solución: no estaban imitando un boceto, ni siquiera un puente: estaban imitando un modelo de negocio. . . <<<

[6] Some additional data:

- The basic load data is as follow: in the central span the dead load amounts up to 438 kN/m, 295 for the truss and 143 for the cables, being 1.12 m the diameter of each main cable. The selfweight can be estimated as 360 kN/m, and the useful maximum load as 216 kN/m. The design wind speed was of 60 m/s.
- The predicted deflections were about 8 m in vertical, and 30 m in horizontal directions.
- 

  The anchorage of main cables is of gravity type: gravity anchorage relies on the mass of the anchorage itself to resist the tension of the main cables, i.e., on friction between foundation and soil. The anchorage sustains a horizontal force of about 1.000 MN. The anchorage body has about 140.000 m^3 of concrete, i.e., about 3.080 MN of weight, and that means a net friction coefficient of about 0.32. 3.080 MN of anchorage weight can be compared with the dead load of half a bridge, i.e., about 850 MN.

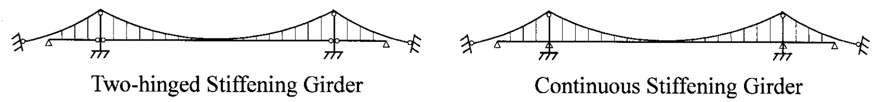
- The dead load causes up to 91% of the stress in main cables.
- In the design of the Akashi Kaikyo Bridge, a safety factor of 2.2 was used using the allowable stress method considering the predominance of the dead load on stress. The main cables used a newly developed high-strength steel wire whose tensile strength is 1770 N/mm^2 and the allowable stress was 804 N/mm^2.
- The overall length of wire in main cables is about 7.5 times the circumference of the Earth.
- For the Akashi Kaikyo Bridge, a global $\frac{1}{100}$ model about 40 m in total length, was tested in a boundary layer wind tunnel laboratory. Together with the verification of the aerodynamic stability of the Akashi Kaikyo Bridge, new findings in flutter analysis and gust response analysis were established from the test results.
- The tower of the Akashi Kaikyo Bridge is 297 m high.
- A dehumidified air-injection system was developed and used on the Akashi Kaikyo Bridge. This system includes wrapping to improve watertightness and the injection of dehumidified air into the main cables.
- Feasibility studies started by Minister of Construction of Japan in 1959...

<<<

[7] To compare Maxwell problem with different counterweight C is not very accurate if we do not account the cost of C. As we will see, the values of C will be very small compared with Q in this very case, and the saving of $\mathcal{V}$ will be very great. For the shake of simplicity, we will compare anyway the values of $\mathcal{V}$ for different values of C, considering the latter free of cost. But it is to be noted that doing so the Maxwell's condition on accounting will not hold. <<<

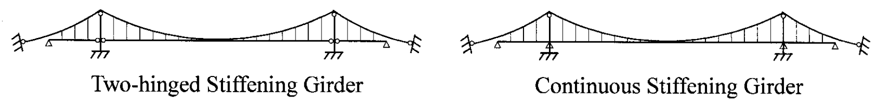
[8] En honor a la verdad, el documento que presenté en el *CIMNE Coffee* contenía un error de bulto: sobrestimé el número de Michell del boceto del AKB en un factor de 6. Con esos primeros resultados el puente resultaba inviable incluso con los aceros de alta gama. Tras el *Coffee* me di cuenta de lo malo que es dejar las cosas para la última hora: había colocado la articulación a la mitad de la luz como en otros casos. Pero aquí, un puente de cuatro apoyos y no de dos, había más posibilidades. La corrección del error trajo buenas noticias: el puente no era tan ''malo''; además la teoría GMMα explica bien ahora por qué se seleccionó la tensión de servicio del acero del cable funicular: no sólo convertía en viable el diseño, acercaba la proporción teórica entre peso propio y carga útil a los estándares de todos los puentes anteriores. . . <<<

[9]



The real AKB is a 2-hinge bridge, and this model is the more accurate for it. The only difference is the anchorage type: in these (self-anchoraged) models the anchorage cost is accounted through the compression volume along all the girder so this kind of accounting does not make any difference when we compare these models among them. <<<

[10]



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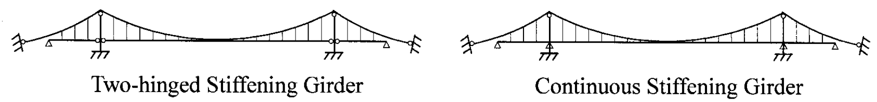
[11]



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[12] Las largas diagonales sometidas a pequeñas compresiones permiten conjeturar que el coste de este diseño será bastante mayor que lo que el valor de $\mathcal{V}$ sugiere: es el coste de

la inestabilidad—o de la complejidad para evitarla. Una simple cercha de 4 km es, a fin de cuentas, una solución teórica. <<<

[13] Although it is customary to speak about the ‘‘oil price’’, this price is for paying the extraction and transportation of oil, it is not in any way for the oil itself. This will be very clear with a simple question: could an oil company make the same quantity of synthetic oil with the money received for extracted oil? Of course it could not! (*reductio ad absurdum* proof: If it will be the case that the company could, then there would be no preoccupation about ‘‘peak-oil’’ and future fuel scarcity as there is nowadays, as the solution would be simple: just to make synthetic oil!. This is the key point stated by Clausius about coal.) <<<

[14] The so named Great Recession of nowadays is precisely the most brilliant experiment that anybody can imagine for proving the Clausius's thesis. (Unfortunately, many persons all over the world are painfully suffering its consequences. . . and to avoid all this sufferings was the main aim of Clausius's booklet: the thermodynamics's laws were enough proving media.) <<<

[15] Surprisingly, optimal solutions for not equivalent problems can have equal properties, e.g., the proportion of the parabolic arch of optimal stress volume in the free load problem is exactly the same that the Prager & Rozvany solution without vertical hangers in the fixed boundary one. . . . <<<

[16] Michell's lemma can be extended in several ways. Firstly, note that the lemma holds also if $k^- = k^+$ independently of the condition $\delta\mathcal{M} = 0$; this is a very restricted case, very useful in prelimanry stages of design although unfortunately it leads to a great confusion about the relationship between fixed boundary and free load classes when it is considered a canonical case; the reason is that with this condition the free load and fixed boundary classes of problems

seem equivalent. Secondly, if both k^+, k^+ are functions we have a non-linear cost; it can be proved that if $\delta\mathcal{M} = 0$, $k^+ = k^+(\mathcal{V}^+)$, $k^- = k^-(\mathcal{V}^-)$, $\partial k^+/\partial\mathcal{V}^+ \geq 0$ and $\partial k^-/\partial\mathcal{V}^- \geq 0$ then the lemma also holds. <<<

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